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Effect of drifting carriers on longitudinal electro-kinetic waves in ion-implanted semiconductor plasmas

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Abstract. We present an analytical study of wave spectra of electro-kinetic waves propagating through semiconductor plasma, whose main constituents are drifting electrons, holes and non-drifting negatively charged colloids. By employing the hydrodynamical model of multi-component plasma, a compact dispersion relation for the same is derived. This dispersion relation is used to study slow electro-kinetic wave phenomena and resultant instability numerically. We find some important modifications in the wave spectra of the slow electro-kinetic branch. It is found that the drift velocities of electrons and holes are responsible for converting two aperiodic modes into periodic ones. The applied dc electric field increases the phase velocities of contra-propagating modes. The amplification coefficients of propagating modes can be optimized by tuning the amplitude of applied electric field and wave number. It is hoped that the results of this investigation should be useful in understanding the wave spectra of slow electro-kinetic waves in ion-implanted semiconductor plasma subjected to a dc electric field along the direction of wave propagation.

PACS. 52.35.Fp Electrostatic waves and oscillations (e.g., ion-acoustic waves) – 72.30.+q High-frequency effects; plasma effects – 61.72.Ww Doping and impurity implantation in other materials – 82.70.Dd Colloids

Recently, the present authors have reported a comprehensive investigation of propagation of new longitudinal electro-kinetic modes ([1]; hereafter referred as paper I) and novel properties introduced due to the presence of negatively charged colloids in semiconductor plasma. We found important modifications in the electro-kinetic branch as well as the existence of new modes of propagation in colloids laden semiconductor plasmas. The results reported in paper I may be useful in understanding the characteristics of longitudinal electro-kinetic waves in colloid laden semiconductor plasmas whose main constituents are non drifting electrons, holes and negatively charged colloids.

Following the work reported in paper I, in the present communication, the authors have focused their attention on the effect of drifting carriers (electrons and holes) on the dispersion and absorption characteristics of longitudinal electro-kinetic waves in ion-implanted semiconductor plasmas.

For this let us consider an ion-implanted semiconductor sample of infinite extent whose main constituents are electrons, holes and negatively charged colloids as discussed in paper I. A static electric field $\mathbf{E}_0$ is imposed on the sample along the negative z-direction, so that now electrons and holes gain drift velocities with average velocities ϑ_{0e} and ϑ_{0h}, along $+z$- and $-z$-directions, respectively.

It is well-known that unless one considers the lowest part of the grain mass spectrum and very low frequency modes, grain dynamics can be safely ignored with respect to electron and hole dynamics. Hence, the medium immersed in an electrostatic field $\mathbf{E}_0$ can be safely treated as a multi-component plasma consisting of drifting electrons and holes, and non-drifting negatively charged colloids under the hydrodynamic limit.

This multi-component plasma system may be described by the component continuity and momentum transfer equations as

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial z}\left(n_\alpha \vartheta_\alpha\right) = 0, \tag{1}$$

$$\frac{\partial \vartheta_{z1\alpha}}{\partial t} + \vartheta_{0\alpha}\frac{\partial \vartheta_{z1\alpha}}{\partial z} = \frac{z_\alpha q_\alpha}{m_\alpha}E_{z1} - \nu_\alpha \vartheta_{z1\alpha} - \frac{\vartheta_{t\alpha}^2}{\rho_{0\alpha}}\frac{\partial \rho_{1\alpha}}{\partial z}. \tag{2}$$

Both the equations and notations are already explained in paper I except $\vartheta_{0\alpha}$ which represents the average drift velocities of electrons ($\alpha = e$) and holes ($\alpha = h$) due to the presence of an electrostatic field $\mathbf{E}_0$ applied along negative z-direction.

Assuming the first order quantities vary as $\exp\left[i\left(\omega t - kz\right)\right]$ (where ω and k are angular frequency and wave number of the propagating mode, respectively), and

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$$
\begin{aligned}
&\omega^4 - i\omega^3\left[\left\{\omega_{Re}\left(k^2\lambda_{De}^2 - 1\right) + i\omega_{Rh}\left(k^2\lambda_{Dh}^2 - 1\right)\right\} - i\left\{k\left(\vartheta_{0e} - \vartheta_{0h}\right)\right\}\right] \\
&- \omega^2\left[\left\{k^2\left(k^2\lambda_{De}^2\lambda_{Dh}^2 - \lambda_{De}^2 - \lambda_{Dh}^2\right)\omega_{Re}\omega_{Rh} - \omega_{pd}^2 + k^2\vartheta_{0e}\vartheta_{0h}\right\} + i\left\{k^3\left(\lambda_{De}^2\omega_{Re}\vartheta_{0h} - \lambda_{Dh}^2\omega_{Rh}\vartheta_{0e}\right) + k\left(\vartheta_{0e}\omega_{Rh} - \vartheta_{0h}\omega_{Re}\right)\right\}\right] \\
&- i\omega\left[k^2\omega_{pd}^2\left(\lambda_{De}^2\omega_{Re} + \lambda_{Dh}^2\omega_{Rh}\right) - ik\omega_{pd}^2\left(\vartheta_{0e} - \vartheta_{0h}\right)\right] \\
&- k^2\left[\left(k^2\lambda_{De}^2\lambda_{Dh}^2\omega_{pd}^2\omega_{Re}\omega_{Rh}^{+}\vartheta_{0e}\vartheta_{0h}\omega_{pd}^2\right) + ik\left(\vartheta_{0h}\lambda_{De}^2\omega_{pd}^2\omega_{Re} + \vartheta_{0e}\lambda_{Dh}^2\omega_{pd}^2\omega_{Rh}\right)\right] = 0 \qquad (5)
\end{aligned}
$$

following the procedure adopted by Steele and Vural [2] and in paper I, the modified dispersion relation for a longitudinal electro-kinetic wave in a medium consisting of drifting electrons and holes, and non-drifting but participating negatively charged colloids is obtained as:

$$
\varepsilon(\omega, k) = 1 + \frac{\omega_{pe}^2}{\left[\left(\omega - k\vartheta_{0e}\right)^2 - i\nu_e\left(\omega - k\vartheta_{0e}\right) - k^2\lambda_{De}^2\omega_{pe}^2\right]} + \frac{\omega_{ph}^2}{\left[\left(\omega + k\vartheta_{0h}\right)^2 - i\nu_h\left(\omega + k\vartheta_{0h}\right) - k^2\lambda_{Dh}^2\omega_{ph}^2\right]} + \frac{\omega_{pd}^2}{\omega^2} = 0. \qquad (3)
$$

If one neglects the average drift velocities of electrons (ϑ_{0e}) and holes (ϑ_{0h}), the above equation reduces to equation (4) of paper I.

Now we shall focus our attention towards the principal point of this communication i.e. instability characteristics of the longitudinal electro-kinetic mode in colloid laden semiconductor plasma media. It is not easy to achieve a fast electro-kinetic mode in the presence of drifting carriers in the medium; hence we shall study the dispersion relation under the slow electro-kinetic mode situation only.

If the phase velocity of the wave is less than the average velocities of electrons and holes, the mode may be termed a slow electro-kinetic mode. Therefore for slow electro-kinetic mode, under collision dominated or low frequency regimes $[\nu_{e,h} \gg (\omega \mp k\vartheta_{0e,h})]$, the dispersion relation reduces to

$$
1 - \frac{\omega_{pe}^2}{\left[k^2\lambda_{De}^2\omega_{pe}^2 + i\nu_e\left(\omega - k\vartheta_{0e}\right)\right]} - \frac{\omega_{ph}^2}{\left[k^2\lambda_{Dh}^2\omega_{ph}^2 + i\nu_h\left(\omega + k\vartheta_{0h}\right)\right]} + \frac{\omega_{pd}^2}{\omega^2} = 0. \qquad (4)
$$

Equation (4) may be written in the form of polynomial in ω as

see equation (5) above.

It was demonstrated very clearly by equation (6a) of paper I that two new modes of propagation are introduced by the presence of colloids in the system. Now it may be inferred by comparing equation (5) and equation (6a) of paper I that the drift introduced due to presence of an external electrostatic field effectively modifies the wave spectra of all the four possible modes.

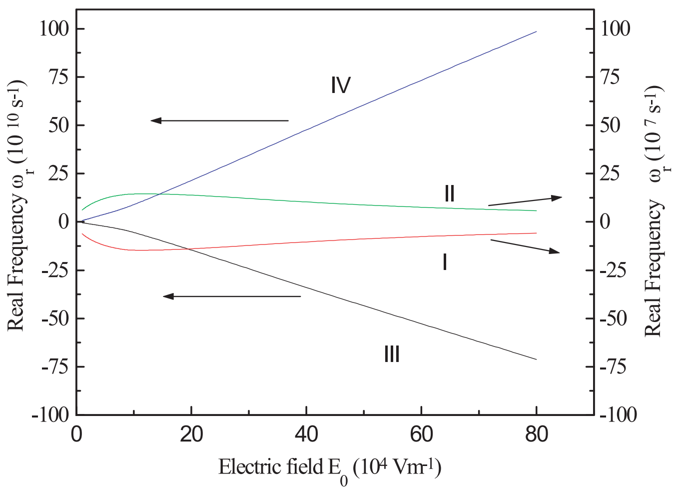

Fig. 1. Variation of real frequency of all four modes with electric field $\mathbf{E}_0$ at $k = 2 \times 10^5$ m^{-1}.

Equation (5) being of fourth degree in complex wave frequency ($\omega = \omega_r + i\omega_i$) with complex coefficients is not easy to solve analytically, and so we solve it numerically using the La Guerre method of finding roots of polynomial. The physical parameters used in the numerical calculations are given in Section 3 of paper I.

The dispersion and absorption characteristics of all possible modes are displayed in Figures 1–7. Figure 1 displays the variations of real parts of frequencies (ω_r) of all the four modes with electric field strength ($\mathbf{E}_0$) at $k = 2 \times 10^5$ m^{-1}. It is found that all the four modes are periodic in nature in the presence of an electric field whereas in the absence of $\mathbf{E}_0$ the first and second existing modes were reported aperiodic in nature ($\omega_r = 0$) in both the presence and absence of charged colloids (paper I). Hence, the presence of an electric field is responsible for periodicity of the first and second modes. It may also be inferred from this figure that the third and fourth modes and first and second modes have exactly opposing response to $\mathbf{E}_0$. Hence, if one propagates along a positive z-direction, the other propagates along a negative z-direction. But in terms of magnitude, both the modes of the same pair (I and II or III and IV) have nearly identical phase constants. With the increment in the value of carrier drift (electric field), magnitude of phase constants of third and fourth modes increase continuously whereas those of first and second modes first increase slightly and then become nearly independent of $\mathbf{E}_0$.

Rapid Note

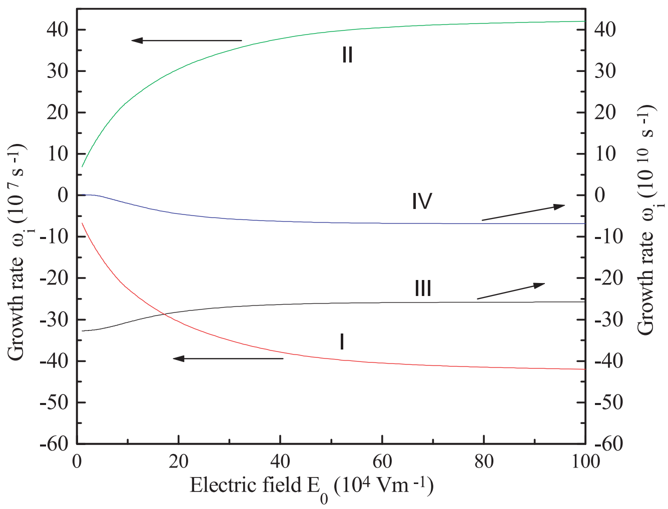

Fig. 2. Variation of growth rates of all four modes with electric field $\mathbf{E}_0$ at $k = 2 \times 10^5$ m^{-1}.

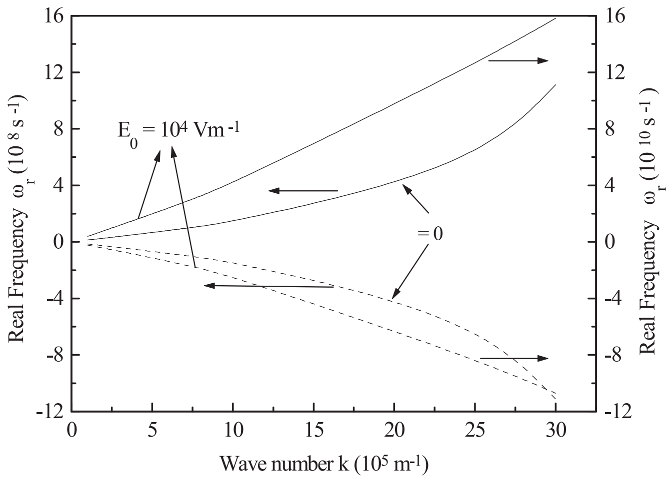

Fig. 4. Variation of real frequency of III and IV periodic modes with wave number k; (- - - -) III mode, (——) IV mode.

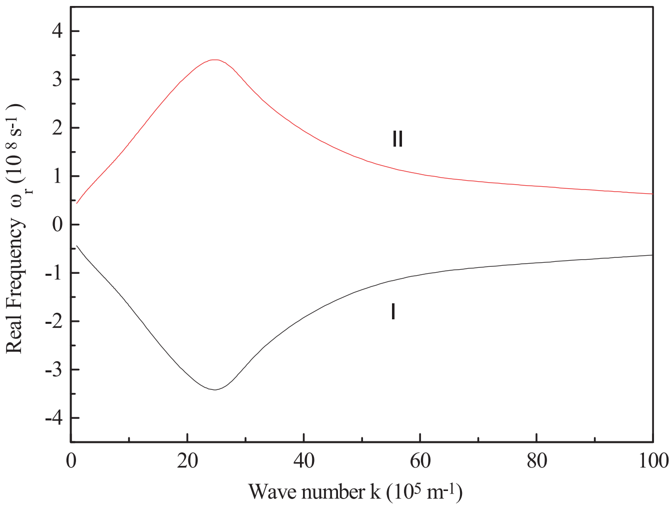

Fig. 3. Variation of real frequency of I and II modes (aperiodic, when $\mathbf{E}_0 = 0$) with wave number k at $\mathbf{E}_0 = 10^4$ Vm^{-1}.

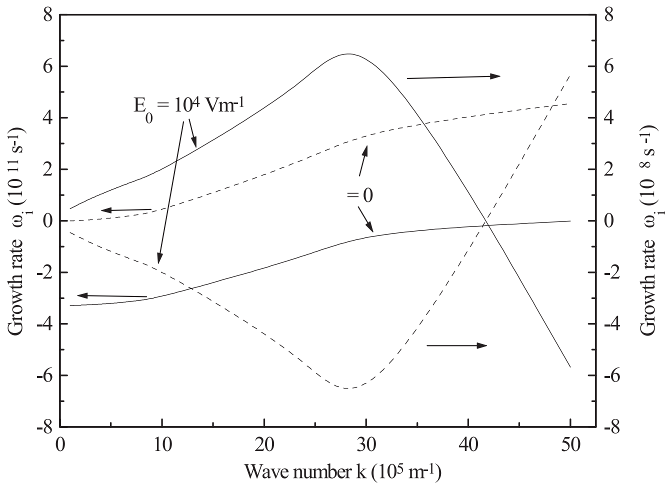

Fig. 5. Variation of growth rate of I and II (aperiodic when $\mathbf{E}_0 = 0$) with wave number k; (- - - -) I mode, (——) II mode.

The dependence of growth rates (ω_i) on electric field $\mathbf{E}_0$ for all the four modes at $k = 2 \times 10^5$ m^{-1} is illustrated in Figure 2. It may be inferred from this figure that three (I, III and IV) modes are growing in nature ($\omega_i < 0$) whereas the second mode is always decaying in nature for the set of parameters used. The gain coefficients of first and fourth modes increase with the increment in the value of electric field $\mathbf{E}_0$ but that of the third mode first decreases slightly and than saturates at higher $\mathbf{E}_0$.

The variations of real frequencies (ω_r) with positive real values of wave number k using $\mathbf{E}_0$ as a parameter are given in Figures 3 and 4. Figure 3 illustrates the ω_r versus k curves of the first and second modes. It is found that these two modes are aperiodic ($\omega_r = 0$) in the absence of $\mathbf{E}_0$ as predicted in paper I. But in the presence of $\mathbf{E}_0$ they become periodic and propagate in opposite directions with equal phase speed. Their phase speeds first increase with k and before reaching a maximum at $k \approx 2 \times 10^5$ m^{-1}. For $k > 2 \times 10^5$ m^{-1}, phase speeds decrease and saturate at higher wave numbers. Figure 4 depicts the variation of ω_r with k for third and fourth modes at $\mathbf{E}_0 = 0$ and $\mathbf{E}_0 = 10^4$ Vm^{-1}. It is found that both the modes are contra-propagating with the fourth one propagating along positive z-direction and the third one along negative z-direction. It may also be inferred that in the absence of an electric field these two modes travel with equal phase speeds in opposite directions. In the presence of an electric field ($\mathbf{E}_0 = 10^4$ Vm^{-1}), they still propagate in opposite directions but with larger phase speeds. Hence, it is confirmed that the drift velocities of electrons and holes in ion-implanted semiconductors are responsible for converting two existing aperiodic modes into periodic modes and for enhancement of phase speeds.

The dependence of growth rates (ω_i) on wave number k in the presence ($\mathbf{E}_0 \neq 0$) and absence ($\mathbf{E}_0 = 0$) of an electric field is depicted in Figures 5 to 7. The variations of growth rates (ω_i) with k for first and second modes are displayed in Figure 5. These are the modes, which are converted into periodic modes due to the presence of

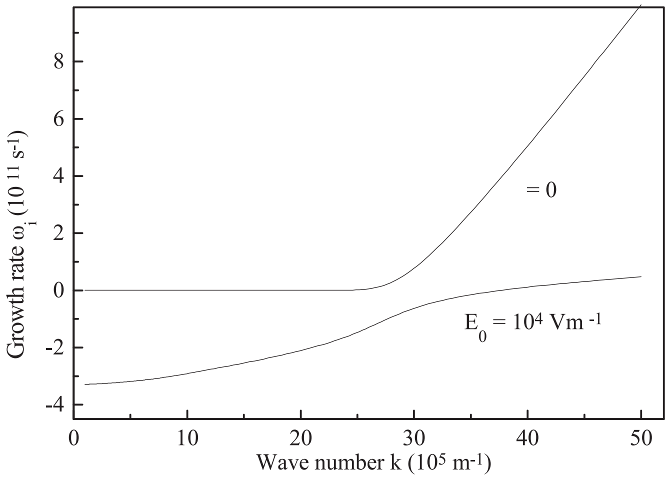

Fig. 6. Variation of growth rate of III mode with wave number k.

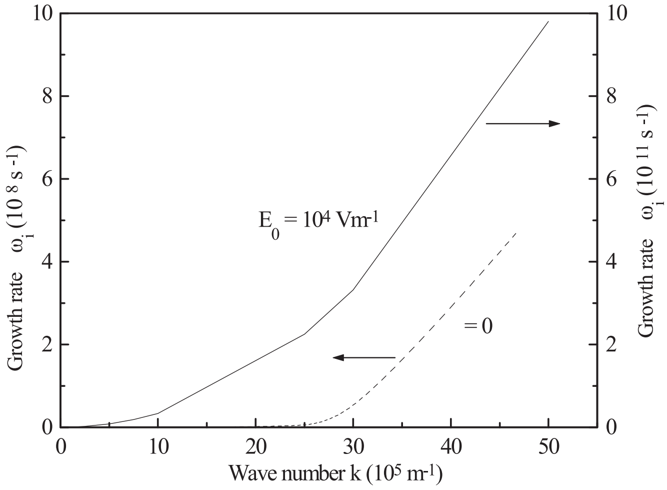

Fig. 7. Variation of growth rate of IV mode with wave number k; (----) $\mathbf{E}_0 = 0$ and (——) $\mathbf{E}_0 = 10^4$ Vm^{-1}.

$\mathbf{E}_0$ in the medium. Figure 5 infers that in the absence of an electric field ($\mathbf{E}_0 = 0$), the first mode is decaying in nature for all values of k but in the presence of $\mathbf{E}_0$, it starts showing amplification characteristics. Its growth rate increases with increasing k, reaches a maximum at $k \approx 3 \times 10^6$ m^{-1} and then starts decreasing. It reaches zero at $k \approx 4 \times 10^6$ m^{-1}and for $k > 4 \times 10^6$ m^{-1}, the mode shows decaying characteristics. In the absence of $\mathbf{E}_0$, the growth rate of second mode initially increases (up to $k < 3 \times 10^6$ m^{-1}), then starts decreasing with the increase in wave number k and becomes zero at $k \approx 4 \times 10^6$ m^{-1}. For $\mathbf{E}_0 = 10^4$ Vm^{-1}, this mode decays with gain coefficient, it first increases with k up to $k \approx 3 \times 10^6$ m^{-1}, then decreases to zero at $k \approx 4 \times 10^6$ m^{-1}. On further increasing k value, the mode changes its nature to amplifying whose growth rate increases sharply with k. Thus both these modes in the presence of carrier drift ($\mathbf{E}_0 \neq 0$) show opposite characters. At $k \approx 4 \times 10^6$ m^{-1}, the gain coefficients of both the modes become zero and beyond this value of the wave number, both exchange their dissipation characteristics.

Figure 6 deals with the third mode of propagation and one may infer that in the absence of an electric field this mode shows decaying nature. Its gain coefficient remains unaffected by wave number k up to $k \approx 3 \times 10^6$ m^{-1} and for $k > 3 \times 10^6$ m^{-1} it increases parabolically. In the presence of $\mathbf{E}_0$, the mode shows amplifying nature. Its growth rate first decreases with k up to $k \approx 3 \times 10^6$ m^{-1} and for $k > 3 \times 10^6$ m^{-1} it becomes independent of the wave number. Thus the presence of a dc electric field is favourable for this mode to achieve amplification in the medium.

Figure 7 displays the growth rate of the fourth mode as a function of wave number k. This mode shows decaying nature in the presence as well as absence of an electric field. The only effect of $\mathbf{E}_0$ on this mode is that it increases the magnitude of the gain coefficient.

To conclude, in this communication, we have investigated the effect of drifting charge carriers on the excitation of low frequency electro-kinetic waves in uniform group IV semiconductors consisting of charged colloids. The important inferences drawn from the study are listed below:

1. by increasing the strength of the electric field, the phase velocity of contra-propagating modes increases;
2. the presence of an electric field is responsible for converting the two aperiodic modes (second and third) into periodic ones. The amplification coefficients of these modes can be optimized by tuning the amplitude of applied electric field and the wave number;
3. the electric field is also found to be favorable in obtaining amplification of the first and fourth modes.

Thus, this fundamental study of wave spectra of the longitudinal electro-kinetic wave in a drifting electron-hole semiconductor plasma embedded with colloids is important for understanding the waves and instability phenomena and can be put to various interesting applications. The slow periodic modes can be favorably employed as a material diagnostic tool owing to their low damping in the presence of an electric field. In the presence of an electric field the fourth mode, because of its increasing absorption in the medium, may be used as a prototype but convenient model for better understanding of the enhanced heating mechanism of electron-hole plasmas in the presence of colloidal particles. Such studies can also provide a compact and less expensive tool for clearer understanding of many laboratory plasmas contaminated with colloids

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